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Mark Scheme (Results)

Summer 2016

Pearson Edexcel GCE in Mechanics 5 (6681/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

  HL Hooke's Law

  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

  RHS, LHS Right hand side, left hand side.

Question Number	Scheme	Marks
1.	Let pv of $B = a\mathbf{i} + b\mathbf{j}$	
	$(0.5\mathbf{i} + \mathbf{j}).((a+1)\mathbf{i} + (b-5)\mathbf{j}) = \frac{1}{2}\text{x}0.4\text{x}5^2$	M1 A1
	$a + 2b = 19$	M1 A1
	$a = 1 - 2\lambda; b = 2 + 3\lambda$	B1
	s olving for a and b	M1
	pv of $B = (-6\mathbf{i} + 12.5\,\mathbf{j})$ m	A1 **7**
	OR	
	$(0.5\mathbf{i} + \mathbf{j}).(-2\lambda\mathbf{i} + 3\lambda\mathbf{j}) = \frac{1}{2}\text{x}0.4\text{x}5^2$	M1 A1
	$-\lambda + 3\lambda = 5$	M1 A1
	$\lambda = \frac{5}{2}$	B1
	pv of $B = (-\mathbf{i} + 5\,\mathbf{j}) + \frac{5}{2}(-2\mathbf{i} + 3\mathbf{j})$	M1
	$= (-6\mathbf{i} + 12.5\,\mathbf{j})$ m	A1 **7**
	<u>Alternative using forces and acceleration</u>	
	Re solving along the wire: $(0.5\mathbf{i} + \mathbf{j}).\frac{1}{\sqrt{13}}(-2\mathbf{i} + 3\mathbf{j}) = 0.4a$	M1
	$\frac{5}{\sqrt{13}} = a$	A1
	$v^2 = u^2 + 2as: \quad 5^2 = 2 \times \frac{5}{\sqrt{13}}s$	M1
	$\frac{5\sqrt{13}}{2} = s$	A1
	$\mathbf{AB} = \frac{5}{2}(-2\mathbf{i} + 3\mathbf{j}) = (-5\mathbf{i} + 7.5\,\mathbf{j})$	B1
	pv of $B = (-\mathbf{i} + 5\,\mathbf{j}) + (-5\mathbf{i} + 7.5\,\mathbf{j})$	M1
	$= (-6\mathbf{i} + 12.5\,\mathbf{j})$	A1 **7**
	Notes	
	First M1 for attempt at using work-energy principle, with usual rules First A1 for a correct unprocessed equation Second M1 for producing an equation in a and b (seen or implied) Second A1 for a correct equation B1 for $a = 1 - 2\lambda$ and $b = 2 + 3\lambda$ seen or implied Third M1 for solving for a and b Third A1 for correct answer (must be a <u>vector</u>)	

Alt	First M1 for resolving along the wire, with usual rules First A1 for a correct acceleration seen or implied Second M1 for a complete method, using *suvat* or calculus, to find the distance along the wire Second A1 for a correct distance B1 for **AB** = (-5**i** + 7.5**j**) seen or implied Third M1 for finding the answer Third A1 for correct answer (must be a vector)	

Question Number	Scheme	Marks
2	$\lambda^2 + 4 = 0$ $\mathbf{r} = \mathbf{A}\cos 2t + \mathbf{B}\sin 2t$ PI: $\mathbf{r} = \mathbf{c}\sin t + \mathbf{d}\cos t$ $\mathbf{r}' = \mathbf{c}\cos t - \mathbf{d}\sin t$ $\mathbf{r}'' = -\mathbf{c}\sin t - \mathbf{d}\cos t$ $-\mathbf{c}\sin t - \mathbf{d}\cos t + 4(\mathbf{c}\sin t + \mathbf{d}\cos t) \equiv 3\sin t\ \mathbf{i}$ $\mathbf{c} = \mathbf{i};\ \mathbf{d} = \mathbf{0}$ GS is $\mathbf{r} = \mathbf{A}\cos 2t + \mathbf{B}\sin 2t + \mathbf{i}\sin t$ When $t = \frac{\pi}{2}, \mathbf{r} = \mathbf{i} - \mathbf{j} \Rightarrow \mathbf{A} = \mathbf{j}$ $\mathbf{v} = -2\mathbf{A}\sin 2t + 2\mathbf{B}\cos 2t + \mathbf{i}\cos t$ When $t = \frac{\pi}{2}, \mathbf{v} = \mathbf{0} \Rightarrow \mathbf{B} = \mathbf{0}$ $\mathbf{r} = \mathbf{i}\sin t + \mathbf{j}\cos 2t$ $\mathbf{v} = \mathbf{i}\cos t - 2\mathbf{j}\sin 2t$	M1 A1 B1 M1 A1 M1 M1 A1 M1 A1 M1 A1 A1 **13**
	Notes	
	First M1 for auxiliary equation First A1 for correct CF (condone omission of **r** =) B1 for correct PI (they may realise **d = 0** which is fine) (condone omission of **r** =) Second M1 for differentiating their PI Second A1 for correct 2nd derivative Third M1 for substituting into the DE Fourth M1 for equating coeffs of sin*t* and cos *t* and finding **c** and **d** Third A1 for correct GS with **r** = Fifth M1 for using conditions to find **A** Fourth A1 for **A= j** Sixth M1 for differentiating **r** to give **v** Fifth A1 for **B = 0** Sixth A1 for **v = i**cos*t* – 2**j**sin2*t*	

Question Number	Scheme	Marks
3(a)	$\mathbf{R} = (2\mathbf{j}-\mathbf{k}) + (\mathbf{i}+\mathbf{k}) + (\mathbf{i}+\mathbf{j}) = (2\mathbf{i}+3\mathbf{j})\text{N}$	M1 A1 (2)
3(b)	Moments about *O*: $(4\mathbf{j}-\mathbf{k})\text{x}(2\mathbf{j}-\mathbf{k}) + (2\mathbf{i}+\mathbf{k})\text{x}(\mathbf{i}+\mathbf{k}) + (3\mathbf{i}+\mathbf{j}+\mathbf{k})\text{x}(\mathbf{i}+\mathbf{j})$ $= \quad -2\mathbf{i} \quad + \quad -\mathbf{j} \quad + \quad -\mathbf{i}+\mathbf{j}+2\mathbf{k}$ $= \quad (-3\mathbf{i}+2\mathbf{k})$ $(\mathbf{i}-\mathbf{j}+\mathbf{k}) \text{ x } (2\mathbf{i}+3\mathbf{j}) + \mathbf{G} = (-3\mathbf{i}+2\mathbf{k})$ $(-3\mathbf{i}+2\mathbf{j}+5\mathbf{k}) + \mathbf{G} = (-3\mathbf{i}+2\mathbf{k})$ $\mathbf{G} = (-2\mathbf{j}-3\mathbf{k})\text{ Nm}$	M1 A3 M1 **A2 ft** A1 A1 (9) **11**
3(b) Alt	Moments about (1, -1, 1): $(-\mathbf{i}+5\mathbf{j}-2\mathbf{k})\times(2\mathbf{j}-\mathbf{k}) + (\mathbf{i}+\mathbf{j})\times(\mathbf{i}+\mathbf{k}) + (2\mathbf{i}+2\mathbf{j})\times(\mathbf{i}+\mathbf{j})$ $= \quad -\mathbf{i}-\mathbf{j}-2\mathbf{k} \quad + \quad \mathbf{i}-\mathbf{j}-\mathbf{k} \quad + \quad \mathbf{0}$ $= \quad -2\mathbf{j}-3\mathbf{k}$ Comparing the 2 systems: $\mathbf{0} + \mathbf{G} = (-2\mathbf{j}-3\mathbf{k})\text{ Nm}$ $\mathbf{G} = (-2\mathbf{j}-3\mathbf{k})\text{Nm}$	M1A2 A3 M1 **A1 ft** A1 (9)
	Notes	
3(a)	M1 for adding the 3 forces together A1 for $(2\mathbf{i} + 3\mathbf{j})$	
3(b)	First M1 consistent use of $\mathbf{r}$ x $\mathbf{F}$, with correct no. of terms First A3 for 3 correct vector products, -1 for each incorrect <u>product</u> (A1A1A0) Second M1 for comparing the rotational effect of the 2 systems *about O*, to give an equation: Their $\Sigma\, \mathbf{r}$ x $\mathbf{F} = \mathbf{G} + (\mathbf{i}-\mathbf{j}+\mathbf{k}) \times$ their $\mathbf{R}$ with correct terms (M0 if term missing) Second **A2 ft,** for the equation ft on their $\mathbf{R}$ and their $\Sigma\, \mathbf{r}$ x $\mathbf{F}$, but no products need to be evaluated. Sixth A1 for a correct equation with all products evaluated Seventh A1 for the answer. Units not needed.	
3(b) Alt	First M1 consistent use of $\Sigma\, \mathbf{r}$ x $\mathbf{F}$, with correct no. of terms, with $\mathbf{r}$ relative to (1, -1 , 1) First A2 for correct terms, -1 each incorrect vector First A3 for 3 correct vector products, -1 for each incorrect <u>product</u> Second M1 for comparing the rotational effect of the 2 systems *about* (1, -1 , 1),to give an equation: $\mathbf{0} + \mathbf{G} =$ their $\Sigma\, \mathbf{r}$ x $\mathbf{F}$ Sixth **A1, ft** on their $\Sigma\, \mathbf{r}$ x $\mathbf{F}$, for a correct equation Seventh A1 for the answer. Units not needed.	

Question Number	Scheme	Marks
4.	$\delta A = 2\pi r \delta x$	M1
	$\delta m = 2\pi r \delta x . \dfrac{M}{2\pi r h}$	M1 A1 (ρ)
	$\left(= \dfrac{M\delta x}{h} \right)$	
	$\dfrac{1}{2}\delta m r^2$	B1
	$\delta I = \dfrac{1}{2}\delta m r^2 + \delta m x^2$	M1 A1
	$= \dfrac{M\delta x}{2h}(r^2 + 2x^2)$	A1
	$I = \int_0^h \dfrac{M}{2h}(r^2 + 2x^2)\,\mathrm{d}x$	**DM1**
	$= \dfrac{M}{2h}\left[(r^2 x + \frac{2}{3}x^3 \right]_0^h$	A1
	$= \dfrac{M}{6}(3r^2 + 2h^2)$	A1 **10**
	Notes	
	First M1 for area of hoop (element) Second M1 for finding the mass of their element by multiplying by the mass per unit area (or by a calculated ρ) First A1 for a correct mass per unit area (appropriate ρ) First B1 for correct MI about diameter of hoop Third M1 for use of parallel axes Second A1 for correct in terms of δm Third A1 for correct MI in terms of x Fourth **DM1** dependent on third M1 for integrating Fourth A1 for a correct expression with correct limits Fifth A1 for a correct answer in any form	
	N.B. The first 8 marks are available for misreads of solid cylinder or cylindrical shell with end(s).	

Question Number	Scheme	Marks
5(a)	$I_L = \frac{4}{3}ma^2 + \frac{1}{3}ma^2 + m(a^2 + (2a)^2)$ $= \frac{20}{3}ma^2$ PRINTED ANSWER	M1 A1 A1 (3)
(b)	$m\begin{pmatrix}0\\a\end{pmatrix} + m\begin{pmatrix}a\\0\end{pmatrix} = 2m\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix} \Rightarrow \bar{x} = \bar{y} = \frac{a}{2}$ $AG = \frac{a}{2}\sqrt{1^2 + 3^2} = \frac{a}{2}\sqrt{10}$ $M(A), \quad 2mg\frac{a}{2}\sqrt{10}\sin\theta = -\frac{20}{3}ma^2\ \ddot{\theta}$ $-\frac{3g\sqrt{10}}{20a}\theta = \ \ddot{\theta}$, for small θ $T = 2\pi\sqrt{\frac{2a\sqrt{10}}{3g}}$	M1 A1 M1 A1 A1 M1 **DM**1 A1 (8) **11**
	Notes	
5(a)	First M1 for a complete method to find MI with correct no. of terms First A1 for a correct expression Second A1 for a correct PRINTED ANSWER. M0 for $4/3ma^2 + 4/3ma^2 + m(2a)^2$	
5(b)	First M1 for attempt to find *both coordinates* of the CM First A1 for correct position (They could find this by inspection) Second M1 for moments about A, with correct no. of terms, and usual rules, in particular the RHS must be dimensionally correct. Second and third A marks for a correct general equation with θ the angle between AG and the vertical, A1 for each side. Third M1 for use of small angle approximation and putting into SHM form. **N.B.** Not available if θ is not the angle between AG and the vertical Fourth **DM**1, dependent on third M1, for $\frac{2\pi}{\omega}$ Fourth A1 for a correct answer in any form	

Question Number	Scheme	Marks
6(a)	$-mg\delta t = (m+\delta m)(v+\delta v)+(-\delta m)(v-u)-mv$ $-mg\delta t = m\delta v + u\delta m$ $-g = \frac{\mathrm{d}v}{\mathrm{d}t} + \frac{u}{m}\frac{\mathrm{d}m}{\mathrm{d}t}$ $m = m_0(5-2t) \Rightarrow \frac{\mathrm{d}m}{\mathrm{d}t} = -2m_0$ $-9.8 - \frac{u}{m_0(5-2t)}.(-2m_0) = \frac{\mathrm{d}v}{\mathrm{d}t}$ $\frac{2u}{(5-2t)} - 9.8 = \frac{\mathrm{d}v}{\mathrm{d}t}$ PRINTED	M1 A2 B1 M1 A1 (6)
(b)	$v = -u\ln(5-2t) - 9.8t(+C)$ $t = 0, v = 0 \Rightarrow C = u\ln 5$ $v = u\ln\frac{5}{(5-2t)} - 9.8t$ $m_0 = m_0(5-2t) \Rightarrow t = 2$ $v = u\ln 5 - 19.6$ (m s^{-1})	M1 A1 M1 A1 M1 A1 (6) **12**
	Notes	
6(a)	First M1 for a general impulse-momentum equation (correct number of terms, excluding any $\delta m\delta v$ terms) First A2 for a correct equation B1 for $m = m_0(5-2t) \Rightarrow \frac{\mathrm{d}m}{\mathrm{d}t} = -2m_0$ Third M1 for substituting for dm/dt to produce an equation in v and t only. Third A1 for PRINTED ANSWER (need 9.8 not g)	
6(b)	First M1 for separating the variables and integrating First A1 for a correct solution (without C) Second M1 for using conditions (or lower limits) Second A1 for a correct particular solution in t Third M1 for putting $m = m_0$ and solving for t. Use of $t = 2$ can imply this M mark Third A1 for correct answer (allow 2g instead of 19.6)	

Question Number	Scheme	Marks
7(a)	$J.2a = \frac{4}{3}ma^2\omega$ $\omega = \frac{3J}{2ma}$	M1 A1 A1 (3)
(b)	$mga\sin 30^\circ = -\frac{4}{3}ma^2\ddot{\theta}$ $\lvert\ddot{\theta}\rvert = \frac{3g}{8a}$	M1 A1 A1 (3)
(c)	$X - mg\cos 60^\circ = ma\ddot{\theta}$ $X - \frac{1}{2}mg = m(-\frac{3g}{8a})a$ $X = \frac{mg}{8}$	M1 A1 A1 M1 A1 (5) **11**
	Notes	
7(a)	First M1 for moment of impulse = Gain in Angular Momentum with all terms dimensionally correct First A1 for a correct equation Second A1 for correct answer	
7(b)	First M1 for taking moments about the axis with all terms dimensionally correct First A1 for a correct equation (θ need not be substituted) Second A1 for a correct positive answer **Alternative:** First M1 for differentiating an energy equation with all terms dimensionally correct First A1 for a correct equation Second A1 for a correct positive answer	
7(c)	First M1 for resolving perpendicular to the lamina, with correct no. of terms, with all terms dimensionally correct First and Second A1's for a correct equation (θ need not be substituted) Second M1 for substituting for (must be dimensionally correct) $\ddot{\theta}$ Third A1 for answer	

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